The Competence Description in Micro 3 says:

Game Theory has become a central analytic tool in much economic theory, e.g. within industrial organization, macroeconomics, international economics, labor economics, public economics and political economics.

The course aims at giving the student knowledge of game theory, non-cooperative as well as cooperative, and its applications in economic models.

The student who successfully completed the course will learn the basic game theory and will be enabled to work further with advanced game theory. The student will also learn how economic problems, involving strategic situations, can be modeled using game theory, as well as how these models are solved. The course intention is thus, that the student through this becomes able to work with modern economic theory, for instance within the areas of within industrial organization, macroeconomics, international economics, labor economics, public economics and political economics.

In the process of the course the student will learn about

- *Static games with complete information*
- *Static games with incomplete information*
- *Dynamic games with complete information*
- *Dynamic games with incomplete information*
- *Basic cooperative game theory.*

For each of these classes of games, the student should know and understand the theory, and learn how to model and analyze some important economic issues within the respective game framework.

More specifically, the students should know the theory and be able to work with both normal and extensive form games. They should know, understand and be able to apply the concepts of dominant strategies, iterative elimination of dominant strategies, as well as mixed strategies. The students should know the central equilibrium concepts in non-cooperative game theory, such as Nash Equilibrium and further refinements: Subgame-Perfect Nash Equilibrium, Bayesian Nash Equilibrium, Perfect Bayesian Equilibrium. They should understand why these concepts are central and when they are used, and be able to apply the relevant equilibrium and solution concepts.

Furthermore, the students should acquire knowledge about a number of special games and the particular issues associated with them, such as repeated games (including infinitely repeated games), auctions and signaling games.

The students should also understand and be able to apply the solution concepts of cooperative game theory, such as the core. Furthermore, the students should also learn the basics of bargaining theory.

To obtain a top mark in the course the student must be able excel in all of the areas listed above.

In view of this, the grading of the exam should take as a point of departure, the short description of the solutions below

MICRO 3 EXAM JUNE 2010
QUESTIONS WITH SHORT ANSWERS
(The answers in this solution are often short/indicative, a good exercise should argue for these answers)

1. (a) Find *all* Nash equilibria in the following game

	L	R
T	$2, 3$	$4, 2$
B	$3, 1$	$1, 2$

Solution: The are no PSNE. The mixed eq can be determined as follows: assume that all pure strategies are played with non-negative probability and assign p as the probability that player 1 plays T and q as the probability that player 2 plays L.

		q	1-q
		L	R
r	T	$2, 3$	$4, 2$
1-r	B	$3, 1$	$1, 2$

Row player is indifferent between playing T and B iff

$$\begin{aligned} 2q + 4(1-q) &= 3q + (1-q) \Leftrightarrow \\ q &= 3/4. \end{aligned}$$

Row player's best response is

$$BR_1(q) = r^*(q) \begin{cases} = 0 \text{ if } q > 3/4 \text{ (strategy B)} \\ \in [0,1] \text{ if } q = 3/4 \text{ (any combination of } T \text{ and } B) \\ = 1 \text{ if } q < 3/4 \text{ (strategy T)} \end{cases}$$

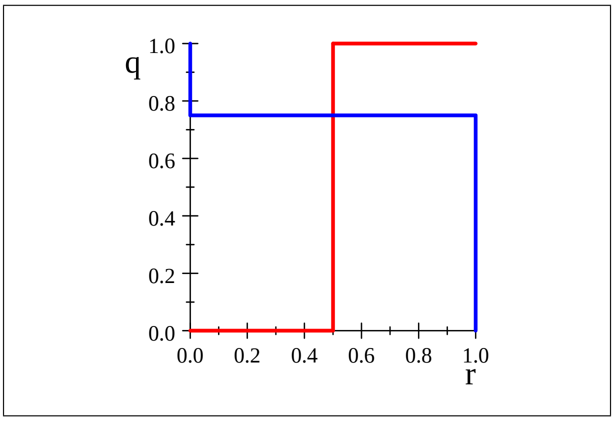
Column player is indifferent between playing L and R iff

$$3r + (1-r) = 2r + 2(1-r),$$

that is, if the row player is mixing with the weight $r = 1/2$. Column player's best response is

$$BR_2(r) = q^*(r) \begin{cases} = 1 \text{ if } r > 1/2 \text{ (strategy L)} \\ \in [0,1] \text{ if } r = 1/2 \text{ (any combination of } L \text{ and } R) \\ = 0 \text{ if } r < 1/2 \text{ (strategy R)} \end{cases}$$

The intersection of BRs is (the BR of Player 1 is in blue, and the BR of player 2 is in red)



Therefore, the mixed strategy equilibrium is $[(1/2, 1/2)(3/4, 1/4)]$, i.e. the row player plays T with prob 1/2, and the column player plays L with prob 3/4.

(b) Consider the following non-cooperative *simultaneous-move* game between 3 players: Player 1 chooses the game matrix (between matrixes (A) and (B) below), and Players 2 and 3 play the respective game. The first number in each cell is the payoff of Player 1, the second number - the payoff of Player 2, and the third one - of Player 3.

(A)

Player 2 \ Player 3	L	R
U	$1, 2, 3$	$5, 1, 1$
D	$2, 5, 1$	$-1, 2, 4$

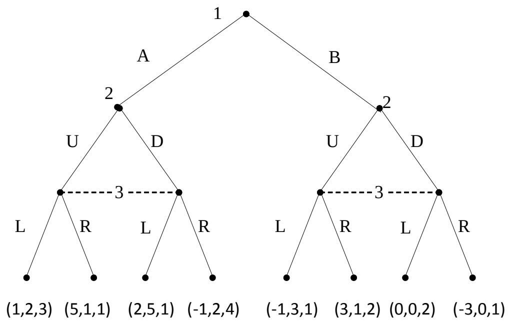
(B)

Player 2 \ Player 3	L	R
U	$-1, 3, 1$	$3, 1, 2$
D	$0, 0, 2$	$-3, 0, 1$

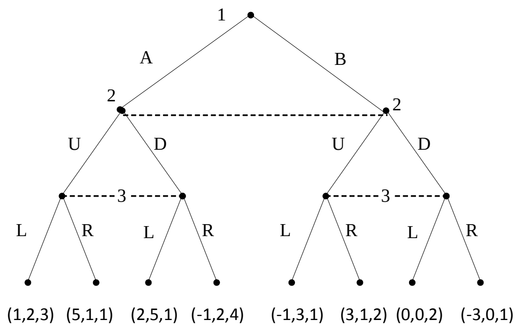
(*)

i. Solve this game by iterated elimination of strictly dominated strategies.
Solution: Strategy (=matrix choice) B is strictly dominated by A for Player 1. Then U is strictly dominated by D for Player 2 and finally L is strictly dominated by R for Player 3. Answer (A, D, R).

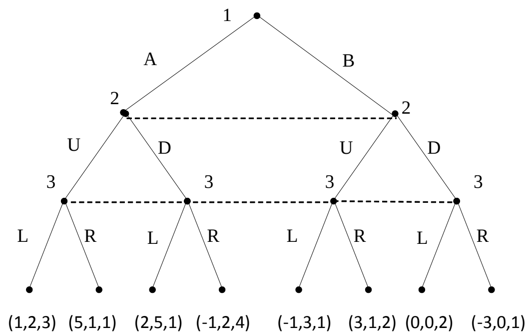
ii. Consider the extensive-form games 1, 2 and 3 below. Which of them (if any) corresponds to the (normal-form) game of subquestion (b)? Explain.



Extensive-form game 1



Extensive-form game 2



Extensive-form game 3

Solution: Game 3. It is a simultaneous-move game, and none of the players knows what the others have done. In Game 1 both Player 2 and Player 3 know what Player 1 has done. In game 2 Player 3 knows what Player 1 have done.

(c) Now assume that *first* Player 1 chooses between the game matrixes (A) and (B) in (*), and *then* Players 2 and 3 observe her choice and *simultaneously* play the respective game.

i. Which of the extensive-form games 1,2 or 3 (above) corresponds to this scenario?
Solution: Game 1 (see above)

ii. How many subgames are in the game that you chose in (c.i)? Find all its subgame perfect Nash equilibria.
Solution: There are 3 subgames including the game itself. The NE in the left proper subgame is (D, R) (look at matrix A), in the right proper subgame is (U,R) (look at matrix B). SPNE is then (B, DU, RR).

iii. In the SPNE that you found in (c.ii), does Player 1 choose the strategy that was dominant for her in the game of subquestion (b)? That is, is the dominance relation "preserved" under the transformation of the game from a simultaneous-move one in (b) to the sequential-move one in (c)? Explain why or why not.

Solution: No she does not. The reason the dominance relation is not preserved is due to the fact that in a sequential-move game the other players can condition their choices on the choice of Player 1. That is, the strategy space is now changed and in the transformed game Player 1 no longer has a dominant strategy.

(d) Can a cooperative game have an empty core? If yes, provide an example and show that the core in your example is empty, if no, explain why.

Solution: Yes it can - think of the simple majority game with 3 players. The value of a 1-player coalition is 0, the value of any 2-player coalition AND of 3-player coalition is 1. Then if there is a core allocation (x_1, x_2, x_3), it should satisfy

$$\begin{aligned}
x_1 + x_2 + x_3 &= v(\{1,2,3\}) = 1 && (1)\\
x_1 + x_2 &\geq v(\{1,2\}) = 1 && (2)\\
x_1 + x_3 &\geq v(\{1,3\}) = 1 && (3)\\
x_2 + x_3 &\geq v(\{2,3\}) = 1 && (4)\\
x_1 &\geq v(\{1\}) = 0 && (5)\\
x_2 &\geq v(\{2\}) = 0 && (6)\\
x_3 &\geq v(\{3\}) = 0 && (7)
\end{aligned}$$

(1),(2),(5)-(7)$\Rightarrow x_3 = 0$,
(1),(3),(5)-(7)$\Rightarrow x_2 = 0$
(1),(4),(5)-(7)$\Rightarrow x_1 = 0$,
but $x_1 + x_2 + x_3 = 1 \Rightarrow$ the core is empty.

2. Two firms $i = 1, 2$ are producing differentiated products and competing in prices. Both firms have constant marginal costs of production c. Before they set prices, they can spend resources on the advertisement, which increases demand by attracting consumers from the competing firm's market. More precisely, if firm i chooses the level of advertisement to be x_i, $i = 1, 2$, then the market demands of both firms become

$$\begin{aligned}
q_1(p_1, p_2, x_1, x_2) &= a + \underbrace{x_1}_{\substack{\text{increase}\\ \text{from own}\\ \text{advertisement}}} - \underbrace{x_2}_{\substack{\text{decrease}\\ \text{from other firm's}\\ \text{advertisement}}} - p_1 + \frac{p_2}{2},\\
q_2(p_1, p_2, x_1, x_2) &= a + x_2 - x_1 - p_2 + \frac{p_1}{2}
\end{aligned}$$

where p_1 and p_2 are the prices set by firms 1 and 2, respectively. An advertisement level of x_i costs $\frac{8}{25}x_i^2$ to firm i. Each firm maximizes its market profit less the advertisement cost

$$\Pi_i = (p_i - c)\, q_i(p_1, p_2, x_1, x_2) - \frac{8}{25}x_i^2, \quad i = 1, 2.$$

The timing of the game is as follows: in the first period both firms simultaneously choose the advertisement levels x_1 and x_2. In the second period firms observe the outcome of the first period and simultaneously set price levels p_1 and p_2, production takes place and the profits get realized.

(a) Consider firms' behavior in the second period of the game. Given the advertisement decisions of the first period, what is the price level $p_i(x_1, x_2)$ that each firm chooses in the NE of the second period? Show that the profit levels of the firms, as functions of x_1 and x_2, are given by

$$\Pi_1 = \left(\frac{2a-c}{3} + \frac{2}{5}(x_1 - x_2)\right)^2 - \frac{8}{25}x_1^2$$

and

$$\Pi_2 = \left(\frac{2a-c}{3} + \frac{2}{5}(x_2 - x_1)\right)^2 - \frac{8}{25}x_2^2,$$

respectively.

Solution: In the second period firm 1 solves the following optimization problem

$$\begin{aligned} & \max_{p_1}(p_1 - c)\, q_1(p_1, p_2, x_1, x_2) - \frac{8}{25}x_1^2 \\ = \; & \max_{p_1}(p_1 - c)\,(a + x_1 - x_2 - p_1 + \frac{p_2}{2}) - \frac{8}{25}x_1^2 \end{aligned}$$

The FOC are

$$(a + x_1 - x_2 - p_1 + \frac{p_2}{2}) - (p_1 - c) = 0$$

which yields the best response function of firm 1

$$p_1 = \frac{a + x_1 - x_2 + \frac{p_2}{2} + c}{2}$$

Similarly, the best response function of firm 2 is

$$p_2 = \frac{a + x_2 - x_1 + \frac{p_1}{2} + c}{2}$$

Thereby in equilibrium we have to solve the system of these two best responses. Substituting the second one into the first one and solving for p_1 we get

$$\begin{aligned} p_1 &= \frac{a + x_1 - x_2 + \frac{a + x_2 - x_1 + \frac{p_1}{2} + c}{4} + c}{2} \Leftrightarrow \\ \frac{15p_1}{8} &= \frac{5\,(a+c)}{4} + \frac{3\,(x_1 - x_2)}{4} \Leftrightarrow \\ p_1 &= \frac{2}{3}(a+c) + \frac{2}{5}(x_1 - x_2) \end{aligned}$$

Similarly,

$$p_2 = \frac{2}{3}(a+c) + \frac{2}{5}(x_2 - x_1)$$

The profit level of firm 1 is

$$\begin{aligned} \Pi_1 &= (p_1 - c)\,(a + x_1 - x_2 - p_1 + \frac{p_2}{2}) - \frac{8}{25}x_1^2 \\ &= \left(\frac{2(a+c)}{3} + \frac{2\,(x_1 - x_2)}{5} - c\right)\left(a + x_1 - x_2 - \frac{2(a+c)}{3} - \frac{2\,(x_1 - x_2)}{5} + \frac{(a+c)}{3} + \frac{(x_2 - x_1)}{5}\right) - \frac{8}{25}x_1^2 \\ &= \left(\frac{2a-c}{3} + \frac{2}{5}(x_1 - x_2)\right)\left(\frac{2a-c}{3} + \frac{2}{5}(x_1 - x_2)\right) - \frac{8}{25}x_1^2 \\ &= \left(\frac{2a-c}{3} + \frac{2}{5}(x_1 - x_2)\right)^2 - \frac{8}{25}x_1^2 \end{aligned}$$

The profit of firm 2 is respectively

$$\Pi_2 = \left(\frac{2a-c}{3} + \frac{2}{5}(x_2 - x_1)\right)^2 - \frac{8}{25}x_2^2$$

(b) Now consider the decisions of the firms in the first period and find the equilibrium advertisement levels of both firms. What are the resulting profit levels of the firms in this equilibrium?

Solution: Now firm 1 solves

$$\max_{x_1} \Pi_1 = \max_{x_1} \left(\frac{2a-c}{3} + \frac{2}{5}(x_1 - x_2) \right)^2 - \frac{8}{25} x_1^2$$

FOC is

$$\begin{aligned} 2 * \frac{2}{5} * \left(\frac{2a-c}{3} + \frac{2}{5}(x_1 - x_2) \right) - \frac{16}{25} x_1 &= 0 \Leftrightarrow \\ \frac{2a-c}{3} + \frac{2}{5}(x_1 - x_2) - \frac{4}{5} x_1 &= 0 \end{aligned}$$

which yields the best response of firm 1 in the first period

$$x_1 = \frac{5}{2}\left(\frac{2a-c}{3}\right) - x_2$$

Similarly, the best response of firm 2 is

$$x_2 = \frac{5}{2}\left(\frac{2a-c}{3}\right) - x_1$$

which yields the following level of advertisements in equilibrium:

$$x_1 = x_2 = \frac{5}{4}\left(\frac{2a-c}{3}\right)$$

The resulting levels of prices

$$p_1 = \frac{2(a+c)}{3} = p_2$$

and the profits of firms in equilibrium are

$$\Pi_1 = \Pi_2 = \left(\frac{2a-c}{3}\right)^2 - \frac{8}{25}\frac{25}{16}\left(\frac{2a-c}{3}\right)^2 = \frac{1}{2}\left(\frac{2a-c}{3}\right)^2$$

(c) Assume that the firms agreed not to advertise (i.e. to set $x_1 = x_2 = 0$).

i. If they stick to this agreement, are they better off or worse off than in (b)? Provide intuition behind your answer.

Solution: Notice that in equilibrium we found above $x_1 = x_2 = \frac{5}{4}\left(\frac{2a-c}{3}\right) > 0$. Therefore the effect of the advertisement on the amount of the output produced by the firms is zero (as $x_1 - x_2 = 0$), while each firm still pays the advertisement costs. Therefore the agreement to set $x_1 = x_2 = 0$ would be beneficial for both firms.

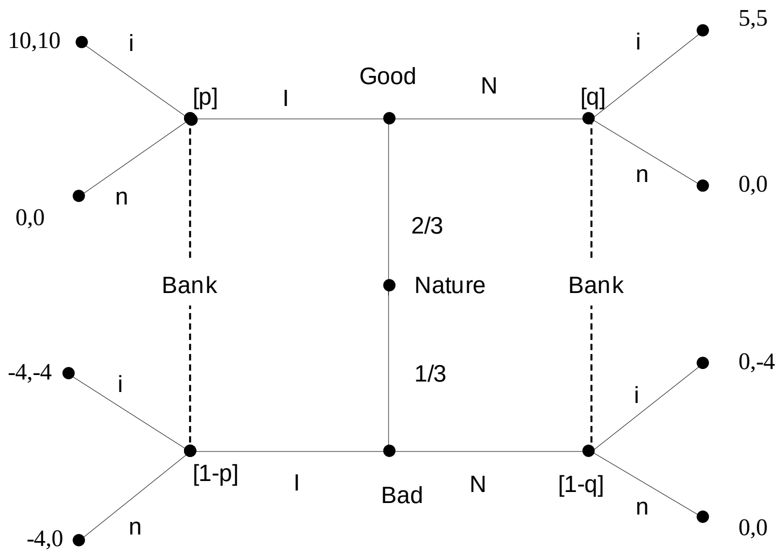
ii. Is this agreement credible (i.e. is any firm willing to deviate, if it assumes that the other one sticks to the agreement)? Explain and provide intuition behind your answer.

Solution: Assume that firm 2 decides to stick to this agreement. Then the BR of firm 1 is

$$x_1 = \frac{5}{2}\left(\frac{2a-c}{3}\right) - x_2 = \frac{5}{2}\left(\frac{2a-c}{3}\right) - 0 = \frac{5}{2}\left(\frac{2a-c}{3}\right) > 0,$$

so firm 1 is better off by choosing positive advertisement level. Therefore the agreement is not credible and cannot be sustained unless it is legally binding. Think of it this way: if the other firm does not advertise, by advertising a small positive Δ you gain quite a bit in market size, but at very low (quadratic) cost. So you would be better off with positive level of advertisement. The same is true for the other firm. However, this becomes a waste of resources for both of you, as your market size depends on the *difference* between your advertisement expenses. So your rationality drives both of you away from the more efficient outcome exactly as in the Prisoners dilemma.

3. A firm is considering starting a new project. Its own capital is not sufficient to finance the project, so it goes to a bank and offers it to be a co-investor on the project. The project is either good (with probability 2/3) or bad (with probability 1/3) and only the firm knows its quality. The timing of the game is as follows: first, nature draws the quality of the project. The firm learns it and decides whether to invest its own money (I) or not (N). Then the bank observes investment decision of the firm and decides whether to invest (i) or not (n). If both firm and bank invest and the project is good, they receive high profits. If only bank invests and the project is good, both the firm and the bank receive moderate profits. The investment of the firm only is not sufficient to yield profits. So, if the firm is the only investor, it gets zero profit in the case of good project. A bad project implies losses for all investors (if any). The following game tree represents the game, where the first number is the firm's payoff and the second number is the bank's one



(a) Find all pooling PBE of this game.

Solution: Notice that the firm never chooses to invest in the case of bad project, so the only possible type of pooling equilibria we can have involves pooling on N. SR3 implies that $q = 2/3$. In the right info set the Bank will choose to invest if

$$\begin{aligned} 5q - 4(1-q) &\geq 0 \Leftrightarrow \\ q &\geq \frac{4}{9} \end{aligned}$$

Therefore the Bank chooses to invest in the right info set. If Bank does not invest in the left info set, nobody wants to deviate. This is associated with the belief that satisfies

$$\begin{aligned} 10p - 4(1-p) &\leq 0 \Leftrightarrow \\ p &\leq \frac{2}{7}. \end{aligned}$$

If the Bank will invest in the left info set, then the firm will deviate for the good project, so it cannot be an equilibrium. Therefore we have a set of pooling equilibria $(NN, ni, p \leq \frac{2}{7}, q = 2/3)$.

(b) Find all separating PBE of this game.

Solution: Assume that the firm chooses to invest for the good project and not to invest for the bad project. SR3 implies that $p = 1$, $q = 0$. Then the Bank will choose to invest in the left info set and not to invest in the right info set. The firm does not have an incentive to deviate for either type of the project, so this is a separating PBE: $(IN, in, p = 1,\ q = 0)$.

Notice that the firm never chooses to invest in the case of bad project, so there are no other separating equilibria.

(c) Formulate signalling requirement 5 and check whether the PBE you found above satisfy requirement 5. Explain your reasoning.

Solution: **DEF:** In a signaling game, the message m_j is dominated for type t_i if there exists another message $m_{j'} \in M$ such that $t_i's$ lowest possible payoff from $m_{j'}$ is higher than the highest possible payoff from m_j, i.e.

$$\min_{a_k \in A} u_S\left(t_i, m_{j'}, a_k\right) > \max_{a_k \in A} u_S\left(t_i, m_j, a_k\right)$$

Signalling Requirement 5 If the information set following m_j is off the equilibrium path and m_j is dominated for type t_i, then (if m_j is not dominated for all types $t_{i'} \in T$) the receiver's belief fulfills $\mu\left(t_i|m_j\right) = 0$.

Intuitively, off equilibrium path the receiver should place zero belief on all the nodes that are reached through dominated (beginning of info set) messages.

The N message is dominated for the bad type of the project. This implies that, in order to satisfy SR5, we should have $p = 1$ off the equilibrium path. This implies that none of our pooling equilibria survives the SR 5. As SR5 "regulates" off-equilibrium beliefs, it has no bite for separating equilibria.